THEORY FOR ELECTROOPTICAL GRATING MODULATORS

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A popular configuration for electrooptical modulators used in integrated optics consists of periodic electrodes placed on the surface of a thin film waveguide made of electrooptical material (Figure 1). When voltages are applied to the electrodes, the thin film guide becomes spatially modulated with periodicity equal to that of the electrodes. Guided light is diffracted after passing through the modulated region. Experiments have been performed with light normally incident[1-2] upon the periodic medium as well as incident at the Bragg angle[3]. The measured results were interpreted with well-known theories applicable either in the Raman-Nath regime[4] or in the Phariseau[5-6] limit. In cases when both limits can not be applied, Klein and Cook[6] devised a numerical solution in which they approximated differential equations by difference equations. The check of the experimental results with these theories has not been satisfactory, especially when the modulation voltage is large.

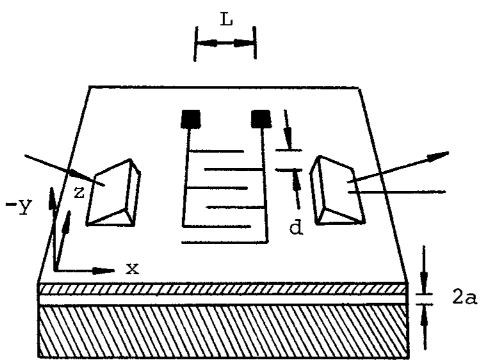

Figure 1

Experiment on an electro-optical grating modulator

These theories suffer two drawbacks. Firstly, they neglect reflections at the two boundaries upon entering and exiting the modulated region. The boundary effects become more important as the modulating voltage is increased. Secondly, they discard terms involving second derivatives. In this report, we propose modifications in theory which remedy these defects. We extend the modal theory by Chu and Tamir[7], which provides the most rigorous approach to the problem of diffraction by a periodically modulated medium.

We assume the model of a slab periodic medium of thickness L (Figure 2) whose permittivity takes the following form:

$$\varepsilon(z) = \varepsilon_r \left[1 - M \cos \frac{2\pi z}{d}\right]$$

where d is the periodicity of the modulation, M is the index of modulation and ε_r is the relative permittivity in the absence of the modulation, i.e. $M = 0$. Under the assumption that $M\lambda/d \ll 1$, the transverse modal function $\phi_\nu(z)$ for both the TE and TM waves in the modulating region satisfy the Mathieu differential equation[7]

$$\frac{d^2\phi_\nu(z)}{dz^2} + (\pi/d)^2 \left(p_\nu - 2q \cos \frac{2\pi}{d} z\right) \phi_\nu(z) = 0$$

with $p_\nu = (d/\pi)^2(\varepsilon_r k^2 - \xi_\nu{}^2)$ and $q = 2M\,\varepsilon_r(d/\lambda)^2$, where the x-component wave numbers ξ_ν are found from p_ν which is in turn determined from the characteristic equations for Mathieu functions. The boundary conditions at $x = 0, L$ are then matched with assumed Floquet wave solutions outside the modulated regions. The problem is then reduced to the solution of a set of properly truncated Matrix equations.

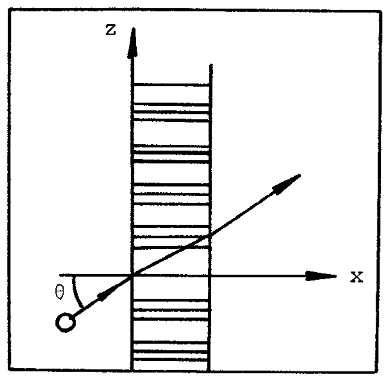

Figure 2

Diffraction by a slab periodic medium

In Figures 3 and 4 we compared our results of the zeroth and the first order diffracted amplitudes with that obtained by Klein and Cook with $Q = 2$ and $Q = 7$ as functions of v, where

$$Q = 2\pi\lambda L/(d^2\sqrt{\varepsilon_r})$$

$$v = \pi L\sqrt{\varepsilon_r}\, M/2.$$

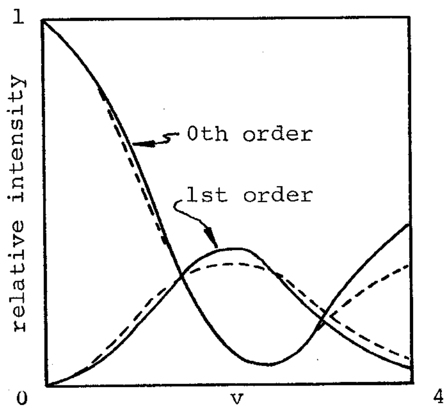

Figure 3

Comparison with results by Klein and Cook (dashed lines) for $Q = 2$

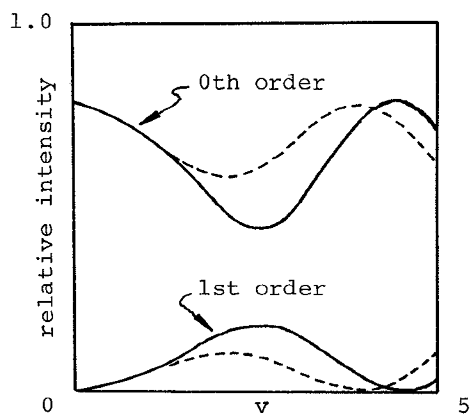

Figure 4

Comparison with results by Klein and Cook (dashed lines) for $Q = 7$

Figure 5 compares our results with data[3] at the first Bragg angle incidence. Note in particular that the maximum value is not unity in contrast to Phariseau's theory which predicts a sinusoidal distribution with unit amplitude. In Figure 6 we compare with the Raman-Nath theory and the experimental results[1] at normal incidence corresponding to $Q = 0.14$.

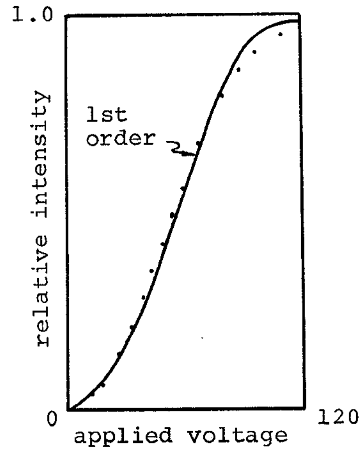

Figure 5

Comparison with experimental data at Bragg angle incidence

Because of the finite dimension of the region of modulation in the $\hat{y}$-direction, diffraction losses out of the electrooptic slab are to be expected that are not present in the infinite slab model. An upper limit to this loss can be calculated by using an integral equation based on a dyadic Green's function for the spatially periodic problem.

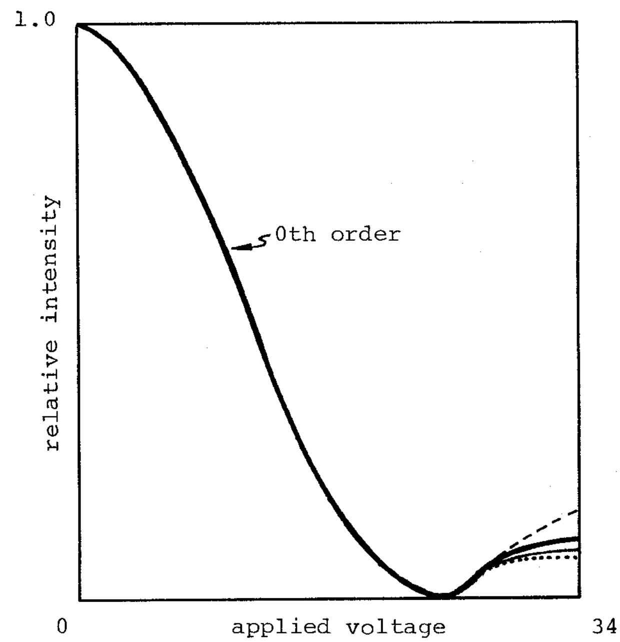

Figure 6

Comparison with the Raman-Nath theory (dashed lines) and with experimental data (dotted lines) at normal incidence

The scattered radiation in the far field from a modulated slab of thickness Δ in the $\hat{x}$-direction and thickness $2a$ in y was calculated with the integral equation and assuming for the first Born approximation the field of a modulated region infinite in y. The surfaces of the electrooptic slab at $y = \pm a$ were assumed to be sufficiently rough that all components of the scattered radiation incident on the boundary would be scattered out of the slab and therefore not detectable at $x = L$. The loss out of the slab of thickness Δ was used to obtain an attenuation coefficient for the radiation propagating within the slab. The attenuation was found to be proportional to M^2 and lead to a maximum power loss of 25% for $v = 4$. Taking this loss into account we see from Figure 6 that the theoretical values are further decreased (represented by thin solid line) and become much closer to the experimental results.

Acknowledgement

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